# Direct Synthesis of Cascaded Quadruplet (CQ) Filters

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*Abstract*—**Previous designs for CQ filters have required matrix rotation operations on the coupling matrix of the canonic form of the cross-coupled filters. This is a rather awkward and not entirely satisfactory process since the theory is not general, requiring the application of equations specific to each order of filter, and in fact has been developed only as far as even order 10. A new direct CQ synthesis has now been discovered having no such limitations. It is shown how the synthesis may be carried out by applying a new equivalent circuit identity to transform a lumped element filter into a cross-coupled CQ filter.**

## I. Introduction

A CQ FILTER CONSISTS OF cascaded groups of 4 cavities or nodes, each with one cross coupling. This is illustrated by the 8th-order coupling diagram of Fig. 1, which contains two CQ sections separated by one normal main coupling, $M_{45}$. Restrictions on the form of transfer function for this type of network are well documented, e.g., [2]. In particular the transmission zeros must be on either the real or imaginary axes, and no complex transmission zeros are allowed. The CQ structure possesses the significant advantage that each CQ section is entirely responsible for producing one transmission zero. This is not the case for other realizations such as the well known canonic structure of Fig. 2 where each cross coupling affects all the transmission zeros, making the filter more difficult to tune. The simpler tunability of CQ filters makes them attractive for commercial applications where cost is a prime consideration.

The most common realization of CQ filters have been as dual mode cavity filters where the cross couplings across 4 nodes takes place between physically adjacent cavities as depicted in Fig. 3. Single-moded realizations are becoming popular also, particularly in the form of cross-coupled combline filters, an example of which is given in Section IV.

Previously the only known analytical (as opposed to numerical) method for designing CQ filters has been by applying matrix rotations to the canonic form of the network, for which synthesis techniques exist [1]–[3].[1] A method for extracting CQ sections directly from the transfer function has now been formulated as described in the following sections.

## II. Direct CQ Synthesis

The purpose of this section is to demonstrate necessary and sufficient conditions for the synthesis of a network as a cascade of CQ sections. Initially the extraction of a CQ section from a

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[1] R. J. Wenzel has drawn attention to the publication of a similar transformation in the field of mechanical filters [6] which has apparently escaped the attention of electrical filter designers during the past two decades. However, [6] does not present design equations.

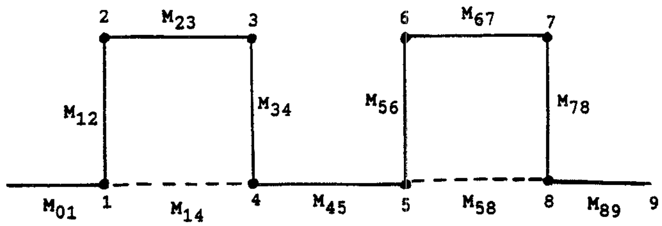



Fig. 1. Coupling diagram of an eighth order CQ filter.

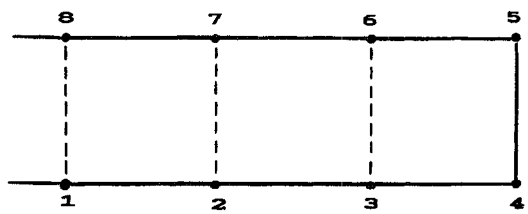



Fig. 2. General canonic form of a cross-coupled filter, $n = 8$.

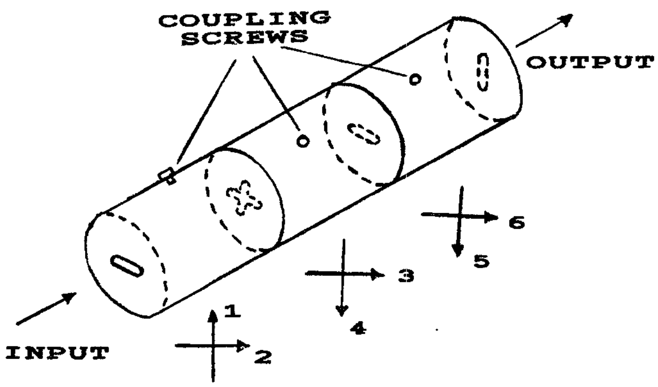



Fig. 3. $TE_{11n}$ dual mode cavity filter, $n = 6$.

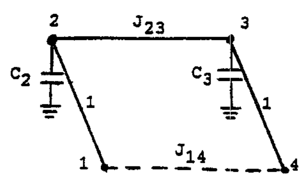



Fig. 4. Partial CQ section equivalent to a Brune or $C$ section.

low pass transfer funcion would seem to be an impossible task because the CQ section is of such high degree and contains several independent parameters. Each of the nodes shown in the example of Fig. 1 has a capacitor to ground as well as main line and possibly cross couplings. However, the situation is not as intractable as it might appear since a shunt capacitor may be extracted from the transfer function, and the remaining portion of the CQ section is the simpler circuit shown in Fig. 4. The capacitor at node 4 may be disregarded, being extracted after the CQ portion has been dealt with.

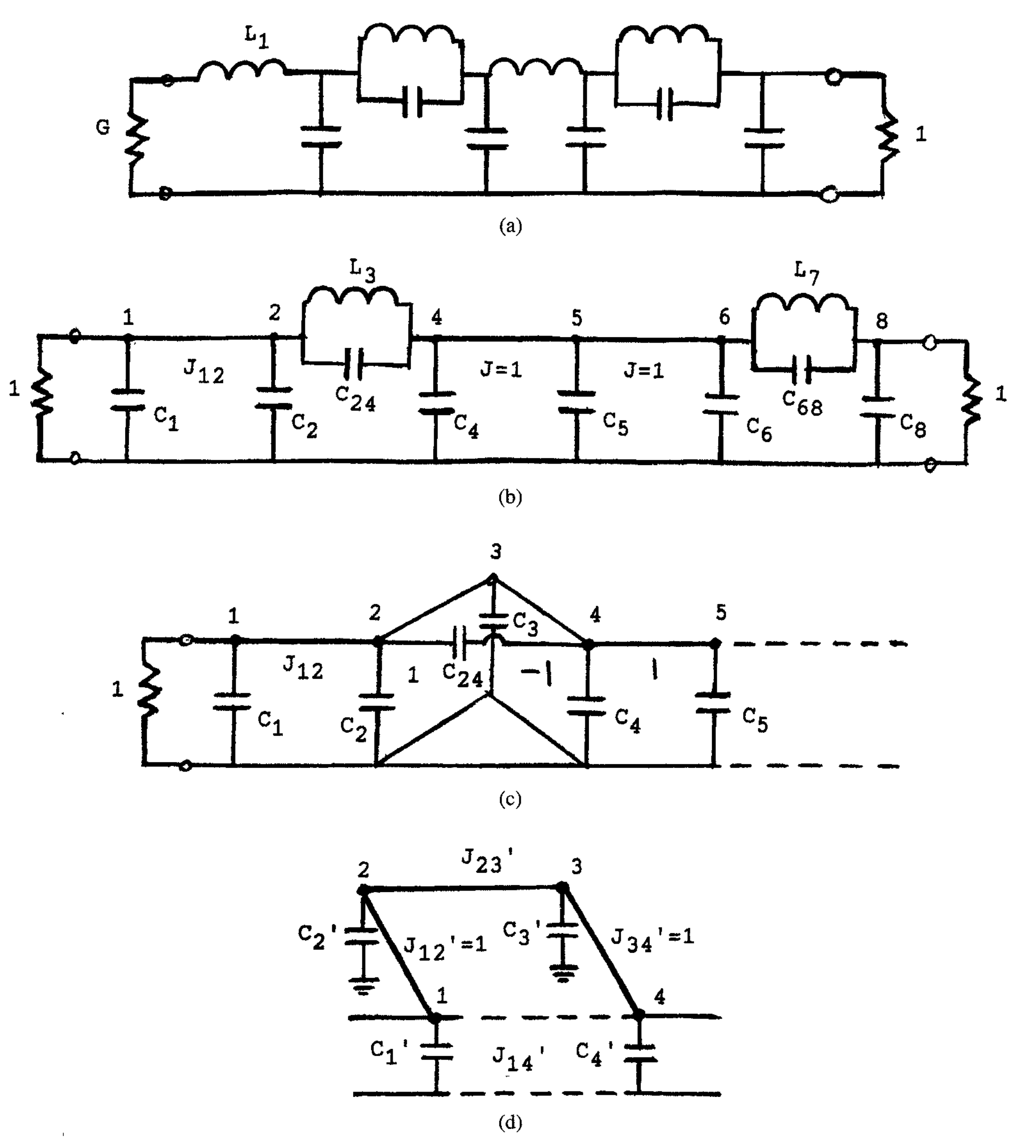



Fig. 5. Transformation of a lowpass filter into CQ format,(a) Lowpass prototype filter $C_1 = J_{12}{}^2 L_1$, $J_{12} = \sqrt{G}$, (b) First stage of transformation,(c) Second stage of transformation, (d) Final conversion into CQ format: $C_1' = C_1$, $C_2' = (C_2 + C_{24})/J_{12}^2$, $C_3' = [(C_2 + C_{24})/C_2]^2 \cdot C_3$, $(C_3 = L_3)$, $C_4' = C_4 + C_2 C_{24}/(C_2 + C_{24})$, $J_{23}' = (C_2 + C_{24})/(C_2 J_{12})$, $J_{14}' = -C_{24} J_{12}/(C_2 + C_{24})$.

In Fig. 4, the lines joining the nodes are admittance inverters, and inverters $J_{12}$ and $J_{34}$ may be set to unity admittance without loss of generality. The transfer matrix of this partial CQ section may be derived as

$$\frac{1/(1 - J_{14}J_{23})}{1 - (\omega^2/\omega_0^2)} \times \begin{bmatrix} C_2\omega/J_{23} & j(C_2C_3\omega^2/J_{23} - J_{23}) \\ j[J_{14}{}^2C_2C_3\omega^2 - (J_{14}J_{23} - 1)^2]/J_{23} & C_3\omega/J_{23} \end{bmatrix} \tag{1}$$

where

$$\omega_0^2 = J_{14}C_2C_3/[(J_{14}J_{23} - 1)J_{23}]. \tag{2}$$

This is a matrix of degree 2 in $\omega$. The proof that it is extractable from the overall transfer matrix of the network follows from the fact that apart from a trivial rotation of the matrix parameters due to having an odd number of admittance inverters in the main path and also the inclusion of an ideal transformer, matrix (1) is exactly that for either a Brune section or $C$-section, e.g., [4]. Appropriate extraction techniques for such sections are well known, and in fact necessary and sufficient conditions which guarantee that such extractions are always possible have been published [4]. The direct extraction process for CQ sections using matrix (1) need not be detailed here since an alternative procedure which requires no new synthesis programming has been obtained, as described below.

## III. Derivation of CQ Filters from Lowpass Filters—A New Network Transformation

In the previous section the existence of a general CQ synthesis was demonstrated. However, rather than having to write a special synthesis program, it has been convenient to

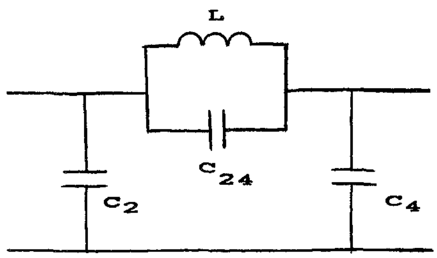



Fig. 6. Lumped element Brune or $C$-section. $C_2C_{24}+C_{24}C_4+C_4C_2=0$.

transform simple cascaded lowpass filters of defined topology into CQ form using an interesting circuit transformation. As an example, the form of circuit to be transformed is shown in Fig. 5(a) for the 8th degree case. This has a 4th ordered attenuation pole at infinity and a pair of second degree finite poles, giving total degree 8. The synthesis of this filter was performed using an existing program for the synthesis of generalized lowpass filters, but may be carried out also using the commercially available program FILSYN [5]. Such programs commence from the generalized rational Chebyshev insertion loss function

$$L = 1 + \varepsilon^2 |f(Z)|^2 \tag{3}$$

as expressed in terms of the transformed variable $Z$ which is related to the normal complex frequency variable $s$ by

$$Z = \sqrt{1 + 1/s^2}. \tag{4}$$

Further details of the formation of such network parameters as the driving point impedances and other network functions required for filter synthesis are given in [3], which also lists other relevant references. The synthesis employs extractions of the various circuit elements expressed directly in the complex variable, with the elements being extracted in the appropriate order to give a circuit in the form shown by the typical example of Fig. 5(a).

The first step in the transformation of this circuit into a CQ filter is to replace each simple series inductor by a cascade of a shunt capacitor flanked on each side by admittance inverters, as shown in Fig. 5(b). At this stage it is convenient also to incorporate the nonunity terminating resistance into one of the inverters. A similar transformation may be performed in the case of the inductors within the pole sections as shown in Fig. 5(c). Here we make use of the identity

$$\begin{bmatrix} 1 & jL\omega \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & j \\ j & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ jC\omega & 1 \end{bmatrix} \begin{bmatrix} 0 & -j \\ -j & 0 \end{bmatrix} \tag{5}$$

where

$$C = L. \tag{6}$$

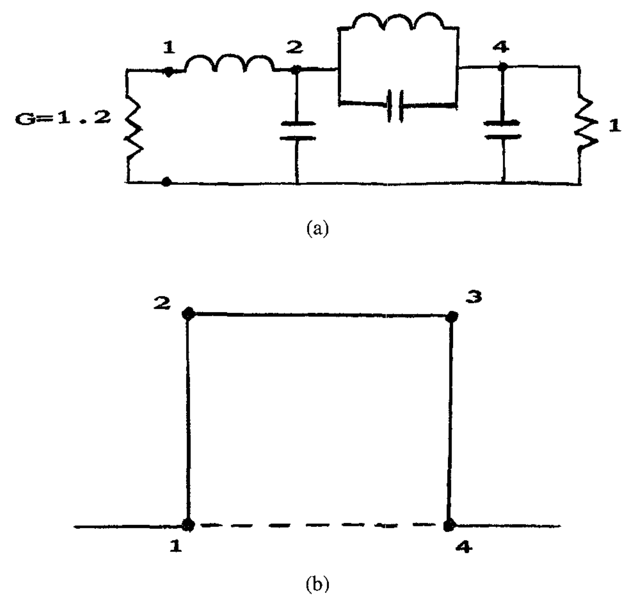



Fig. 7. (a) Lumped element $n = 4$ filter: $G = 1.2$, $L_{12} = 0.767311$, $C_{20} = 1.04060$, $L_{24} = 0.991995$, $C_{24} = 0.252017$, (pole $f_{24} = 2.0$), $C_{40} = 0.707107$, (b) CQ realization: $C_1 = C_4 = 0.920773$, $C_2 = C_3 = 1.380064$, $J_{12} = J_{34} = 1$, $J_{14} = -0.166702$, $J_{23} = 1.076724$.

(The exact equality of (6) would be modified by a factor $J^2$ if the immittance inverters were of admittance $J$ rather than unity, i.e. $C = J^2 L$, giving the correct dimensional relationship).

It is very important to have one of the inverters in (5) have negative admittance to avoid a 1: $-1$ transformer. In the case of the single series inductors of step (a) to (b) such transformers are of no consequence since they do not affect the amplitude of the transfer function.

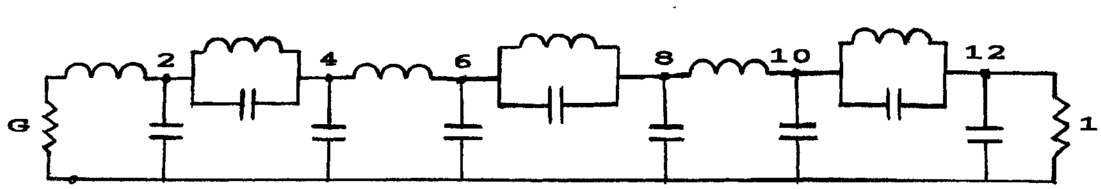
The circuit section between nodes 1 and 4 of Fig. 5(c) has the admittance matrix

$$\begin{bmatrix} C_1 s & -J_{12} & 0 & 0 \\ -J_{12} & (C_2 + C_{24})s & -1 & -C_{24}s \\ 0 & -1 & C_3 s & 1 \\ 0 & -C_{24}s & 1 & (C_{24} + C_4)s \end{bmatrix} \tag{7}$$

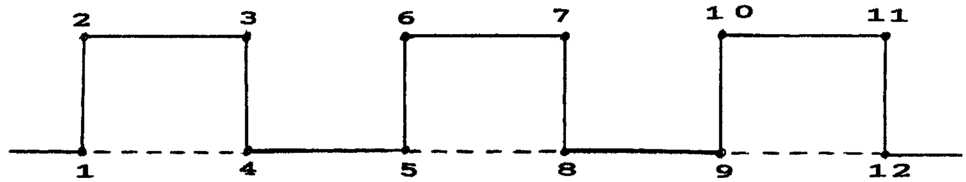
where the complex frequency variable s is used rather than $j\omega$. In order to eliminate the 24 coupling row 2 is multiplied by $C_{24}/(C_2 + C_{24})$ and added to row 4, and a similar operation applied to columns 2 and 4. The $J_{12}$ entries are made unity by multiplying row 2 and column 2 by $1/J_{12}$, giving the equivalent admittance matrix

$$\begin{bmatrix} C_1 s & -1 & 0 & \frac{-C_{24}J_{12}}{(C_2+C_{24})} \\ -1 & \frac{(C_2+C_{24})s}{J_{12}^2} & \frac{-1}{J_{12}} & 0 \\ 0 & \frac{-1}{J_{12}} & C_3 s & 1 - \frac{C_{24}}{(C_2+C_{24})} \\ \frac{-C_{24}J_{12}}{(C_2+C_{24})} & 0 & 1 - \frac{C_{24}}{(C_2+C_{24})} & \left(C_4 + \frac{C_2 C_{24}}{(C_2+C_{24})} s\right) \end{bmatrix}. \tag{8}$$

The process of making the off-diagonal elements 12 and 34 equal to $-1$ is completed by multiplying row and column 3




(a)




(b)

Fig. 8. (a) Lumped element $n = 12$ filter: $G = 1.2$, $L_{12} = 0.839339$, $L_{20} = 1.124050$, $L_{24} = 0.657756$, $C_{24} = 1.149581$, (pole $f_{24} = 1.15$), $C_{40} = 1.542449$, $L_{46} = 1.776032$, $L_{60} = 2.960430$, $L_{68} = 3.155535$, $C_{68} = -0.646742$, (real axis $f_{68} = 0.70$), $C_{80} = 2.954255$, $L_{8,10} = 1.759905$, $C_{10,0} = 1.677657$, $L_{10,12} = 0.754420$, $C_{10,12} = 0.784233$, (pole $f_{10,12} = 1.30$), $C_{12,0} = 0.473057$ (b) CQ realization: $C_1 = 1.007207$, $C_2 = 1.894693$, $C_3 = 2.691121$, $C_4 = 2.110785$, $C_5 = 1.776032$, $C_6 = 2.313688$, $C_7 = 1.927406$, $C_8 = 2.126731$, $C_9 = 1.759905$, $C_{10} = 2.461989$, $C_{11} = 1.624722$, $C_{12} = 1.007519$, $J_{12} = J_{34} = J_{45} = J_{56} = J_{78} = J_{89} = J_{9,10} = J_{11,12} = 1$, $J_{14} = -0.553873$, $J_{58} = 0.279528$, $J_{9,12} = -0.318577$.

by the factor $(C_2 + C_{24})/C_2$. The 23 terms could have been transformed to unity only by introducing a multiplication factor to row and column 4 which would change the admittance looking to the right, an undesirable complication.

The final step is the necessary and rather interesting one of multiplying row and column 4 by $-1$, which gives the 34 terms the correct negative sign and also changes the sign of the 14 terms. The final matrix is given below, and the resulting CQ section shown in Fig. 5(d)

$$\begin{bmatrix} C_1 s & -1 & 0 & \frac{C_{24}J_{12}}{(C_2+C_{24})} \\ -1 & \frac{(C_2+C_{24})s}{J_{12}^2} & -\left(\frac{C_2+C_{24}}{J_{12}C_2}\right) & 0 \\ 0 & \frac{-(C_2+C_{24})}{J_{12}C_2} & \frac{(C_2+C_{24})^2}{C_2^2}C_3 s & -1 \\ \frac{C_{24}J_{12}}{(C_2+C_{24})} & 0 & -1 & \left(C_4 + \frac{C_2C_{24}}{(C_2+C_{24})}s\right) \end{bmatrix}. \tag{9}$$

Note that if $C_{24}$ is positive, corresponding to an attenuation pole, then the 14 term correctly represents a negative admittance inverter, whereas if $C_{24}$ is negative, corresponding to a real axis pole, then the cross coupling inverter is positive. Note that the synthesis of the lowpass prototype filter does indeed produce negative element values for the capacitor $C_{24}$ in the case of $C$-sections where the transmission zero lies on the real axis of the complex frequency plane, a fact which may be verified by carrying out the synthesis using Filsyn [5] for example.

The transformation is applied to each pole-producing section of the original lowpass filter, e.g., to nodes numbered 5, 6, and 8 in Fig. 5(b), resulting in the complete CQ filter.

At this point it is interesting to return to consideration of the basic $C$ or Brune section of Fig. 4 which is the same as Fig. 5(d) with $C_1'$ and $C_4'$ set to zero. The condition that the Pi network of Fig. 6 is a $C$/Brune section is given by

$$C_2C_{24} + C_{24}C_4 + C_4C_2 = 0 \tag{10}$$

which is the well-known cyclic double-product zero condition on the three capacitors. This is easily demonstrated for example by examining the expressions for the capacitors given in [4, (20)].

Application of (10) to the equation for $C_4'$ given in Fig. 5(d) leads to

$$C_4' = C_4 + C_2C_{24}/(C_2 + C_{24}) = 0 \tag{11}$$

$C_1'$ may be considered to be pre-extracted and may be set to zero also. This demonstrates that the circuit of Fig. 4 does indeed satisfy a necessary condition for representing a $C$/Brune section.

## IV. Numerical and Experimental Results

The element values have been compared to those obtained using matrix rotations, with identical results. The theory has been checked also by direct analysis of the derived CQ networks.

The first example is the lowest order case of $n = 4$. The result for the synthesis of a lowpass filter having a 1.2 VSWR ripple bandedge at 1 and normalized pole frequency at 2 is given in Fig. 7(a). Since this is an even-ordered circuit it has unequal terminating resistors and is physically asymmetric. Applying the equations for the CQ transformation as given in Fig. 5 leads to the *symmetrical* circuit of Fig. 7(b), which is a particularly gratifying result for the theory.

As stated earlier no CQ filter for even degree higher than 10 has been published, and it is interesting now to demonstrate

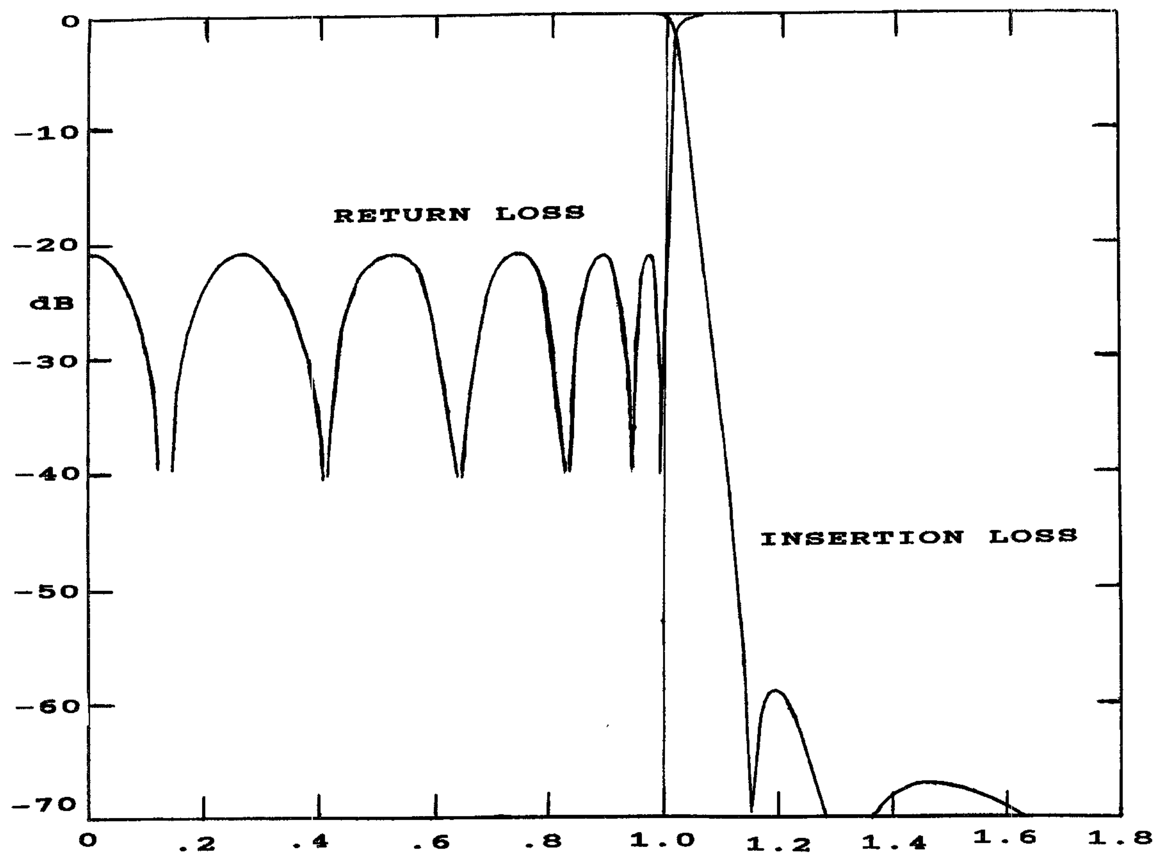



Fig. 9. Analysis of the $n = 12$ CQ filter of Fig. 8(b).

such an example for $n = 12$. The example also has a VSWR of 1.2 (return loss 20.83 dB), with real frequency (imaginary axis) poles at $\pm j$ 1.15 and $\pm j$ 1.3 plus real axis poles at ±0.7. The latter flattens the group delay over the central portion of the passband. The normal lumped element filter is shown in Fig. 8(a), with the conversion to a CQ filter given in Fig. 8(b). The direct analysis of the CQ circuit of Fig. 8(b) is shown in Fig. 9, indicating the exact Chebyshev response with the 6 passband zeros in $0 < \omega' < 1$ and the equiripple response at the 20.83 dB return loss level. The delay variation over the central 50% of the passband is 4% compared with 18% for the same filter without the real axis zero.

Transformations of the lowpass prototypes into bandpass filters have been described extensively in the literature, e.g., [1]. A 12-pole CQ filter with pole locations consisting of a degenerate pair of real frequency poles at $\pm j$ 1.4 and real axis poles at ±0.5 has been realized in combline form with passband edges at 7880 and 8425 MHz. The filter met the specified rejection specifications of >85 dB below 7700 MHz and >40 dB above 8520 MHz, with insertion loss of <1 dB and phase linearity <8° in the 7900–8400 MHz band. The analysed phase linearity is 6°, which was realized in practice.

## V. Conclusion

A new synthesis of CQ filters has been demonstrated which does not rely on conversion of "canonic" type networks using rotation matrices, for which only formulas for certain specific cases have been known. Moreover the new synthesis may be realized by applying a simple network transformation to lumped element networks of prescribed topology which may be derived using existing standard synthesis software.

The paper gives even degree examples of CQ filters up to 12th order (there is no limitation on such order), and it should be pointed out that the theory is equally applicable to filters of odd degree.

The theory is easily extended to the synthesis of cascade trisection (CT) filters where the cross couplings are across 3 nodes. Trisections each realize a pole on one side of the passband of a bandpass filter, giving general asymmetric characteristics.

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Dr. Levy has been involved in many MTT Society activities, including Editor of the IEEE TRANSACTIONS ON MICROWAVE THEORY AND TECHNIQUES from 1986 to 1988. He has been Chairman of the Central New England and San Diego MTT Chapters, and was Vice-Chairman of the Steering Committee for the 1994 IEEE MTT-S International Microwave Symposium.